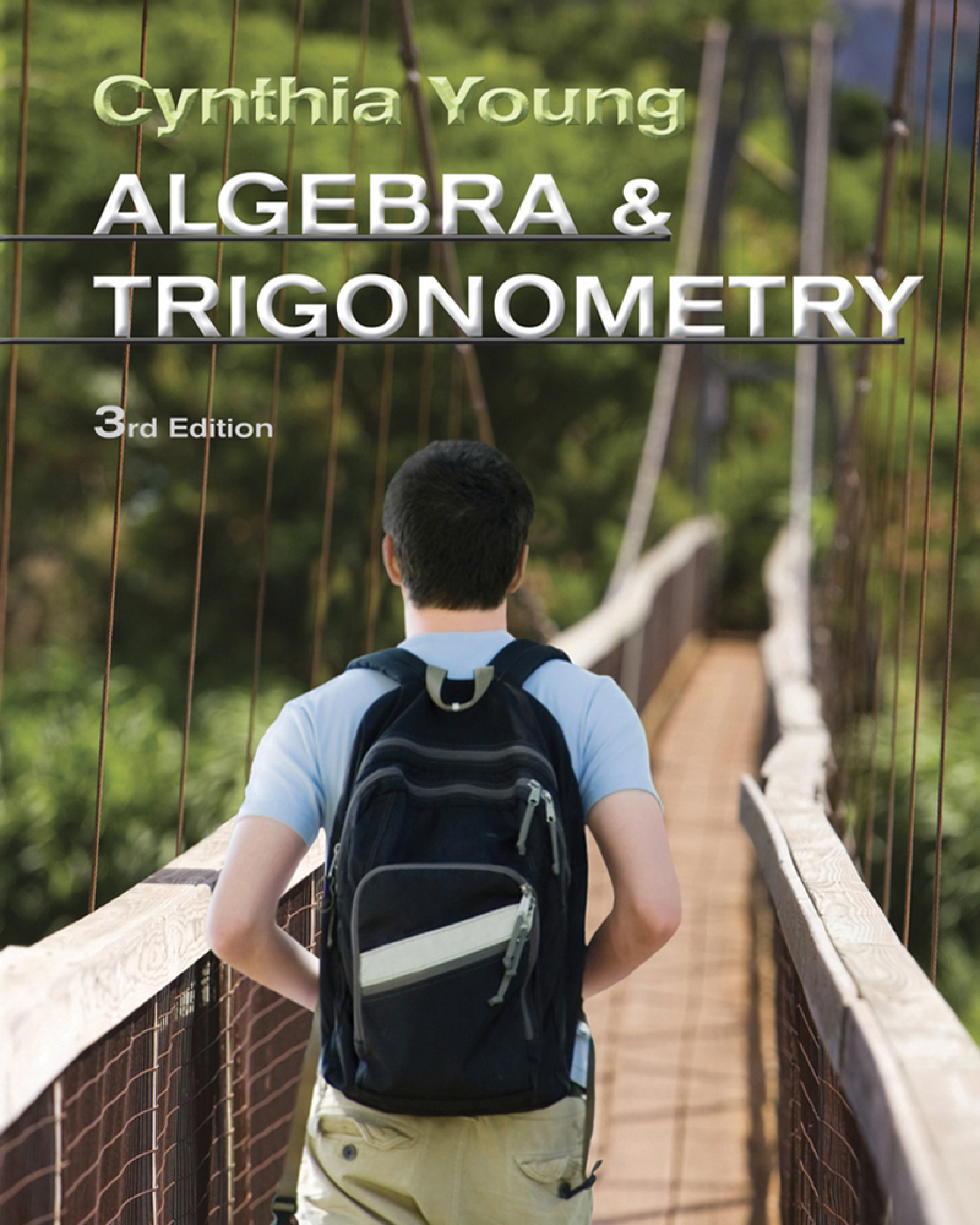
Cynthia Young

ALGEBRA & TRIGONOMETRY

3rd Edition



WileyPLUS

WileyPLUS is a research-based online environment for effective teaching and learning.

WileyPLUS builds students' confidence because it takes the guesswork out of studying by providing students with a clear roadmap:

- what to do
- how to do it
- if they did it right

It offers interactive resources along with a complete digital textbook that help students learn more. With *WileyPLUS*, students take more initiative so you'll have greater impact on their achievement in the classroom and beyond.

Now available for Blackboard

For more information, visit www.wileyplus.com

Algebra and Trigonometry

Third Edition

Algebra and Trigonometry

Third Edition

CYNTHIA Y. YOUNG | *Professor of Mathematics*
UNIVERSITY OF CENTRAL FLORIDA

WILEY

PUBLISHER	Laurie Rosatone
ACQUISITIONS EDITOR	Joanna Dingle
PROJECT EDITOR	Jennifer Brady
ASSOCIATE CONTENT EDITOR	Beth Pearson
EDITORIAL ASSISTANT	Elizabeth Baird
SENIOR PRODUCTION EDITOR	Kerry Weinstein/Ken Santor
DESIGNER	Madelyn Lesure
OPERATIONS MANAGER	Melissa Edwards
ILLUSTRATION EDITOR	Sandra Rigby; Electronic illustrations provided by Techsetters, Inc.
SENIOR PHOTO EDITOR	Jennifer MacMillan
COVER DESIGN	Madelyn Lesure
COVER PHOTO	Front Cover: Combined image: Ty Milford/Masterfile and Leslie Banks/iStockphoto Back Cover: Rustem Gurler/iStockphoto

"Modeling Our World" image: © David Woodfall/Getty Images

This book was set in 10/12 Times by MPS Limited, and printed and bound by Quad/Graphics, Inc.
The cover was printed by Quad/Graphics, Inc.

This book is printed on acid free paper. ∞

ISBN: 978-0-470-64803-2
BRV ISBN: 978-1-118-12930-2

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

The Wiley Faculty Network

The Place Where Faculty Connect

The Wiley Faculty Network (WFN) is a global community of faculty connected by a passion for teaching and a drive to learn, share, and collaborate. Whether you're seeking guidance, training, and resources or simply looking to re-energize your course, you'll find what you need with the WFN. The WFN also partners with institutions to provide customized professional development opportunities. Connect with the Wiley Faculty Network to collaborate with your colleagues, find a Mentor, attend virtual and live events, and view a wealth of resources all designed to help you grow as an educator.

Attend

Discover innovative ideas and gain knowledge you can use.
Learn from instructors around the world, as well as recognized leaders across disciplines. Join thousands of faculty just like you who participate in virtual and live events each semester. You'll connect with fresh ideas, best practices, and practical tools for a wide range of timely topics.

View

Explore your resources and development opportunities.
See all that is available to you when you connect with the Wiley Faculty Network. From Learning Modules and archived Guest Lectures to faculty-development and peer-reviewed resources, there is a wealth of materials at your fingertips.

Collaborate

Connect with colleagues—your greatest resource.
Tap into your greatest resource—your peers. Exchange ideas and teaching tools, while broadening your perspective. Whether you choose to blog, join interest groups, or connect with a Mentor—you've come to the right place!

Wiley Faculty Network

Embrace the art of teaching – Great things happen where faculty connect!

Web: **www.WhereFacultyConnect.com**

EMAIL: **FacultyNetwork@wiley.com**

PHONE: 1-866-4FACULTY (1-866-432-2858)

For Christopher and Caroline

About the Author

Cynthia Y. Young is a native of Tampa, Florida. She currently is a Professor of Mathematics at the University of Central Florida (UCF) and the author of *College Algebra*, *Trigonometry*, *Algebra and Trigonometry*, and *Precalculus*. She holds a B.A. degree in Secondary Mathematics Education from the University of North Carolina (Chapel Hill), an M.S. degree in Mathematical Sciences from UCF, and both an M.S. in Electrical Engineering and a Ph.D in Applied Mathematics from the University of Washington. She has taught high school in North Carolina and Florida, developmental mathematics at Shoreline Community College in Washington, and undergraduate and graduate students at UCF. Dr. Young's two main research interests are laser propagation through random media and improving student learning in STEM. She has authored or co-authored over 60 books and articles and been involved in over $2.5M in external funding. Her atmospheric propagation research was recognized by the Office of Naval Research Young Investigator award, and in 2007 she was selected as a Fellow of the International Society for Optical Engineers. She is currently the co-director of UCF's EXCEL program whose goal is to improve the retention of STEM majors.

Although Dr. Young excels in research, she considers teaching her true calling. She has been the recipient of the UCF Excellence in Undergraduate Teaching Award, the UCF Scholarship of Teaching and Learning Award, and a two-time recipient of the UCF Teaching Incentive Program. Dr. Young is committed to improving student learning in mathematics and has shared her techniques and experiences with colleagues around the country through talks at colleges, universities, and conferences.

Dr. Young and her husband, Dr. Christopher Parkinson, enjoy spending time outdoors and competing in Field Trials with their Labrador Retrievers. *Laird's Cynful Wisdom* (call name "*Wiley*") is titled in Canada and currently pursuing her U.S. title. *Laird's Cynful Ellegance* (call name "*Ellie*") was a finalist in the Canadian National in 2009 and is retired (relaxing at home).

Dr. Young is pictured here with Ellie's 2011 litter of puppies!

Bonnie Farris

Preface

As a mathematics professor I would hear my students say, "I understand you in class, but when I get home I am lost." When I would probe further, students would continue with "I can't read the book." As a mathematician I always found mathematics textbooks quite easy to read—and then it dawned on me: don't look at this book through a mathematician's eyes; look at it through the eyes of students who might not view mathematics the same way that I do. What I found was that the books were not at all like my class. Students understood me in class, but when they got home they couldn't understand the book. It was then that the folks at Wiley lured me into writing. My goal was to write a book that is seamless with how we teach and is an ally (not an adversary) to student learning. I wanted to give students a book they could read without sacrificing the rigor needed for conceptual understanding. The following quote comes from a reviewer of this third edition when asked about the rigor of the book:

> *I would say that this text comes across as a little less rigorous than other texts, but I think that stems from how easy it is to read and how clear the author is. When one actually looks closely at the material, the level of rigor is high.*

Distinguishing Features

Four key features distinguish this book from others, and they came directly from my classroom.

Parallel Words and Math

Have you ever looked at your students' notes? I found that my students were only scribbling down the mathematics that I would write—never the words that I would say in class. I started passing out handouts that had two columns: one column for math and one column for words. Each Example would have one or the other; either the words were there and students had to fill in the math, or the math was there and students had to fill in the words. If you look at the Examples in this book, you will see that the words (your voice) are on the left and the mathematics is on the right. In most math books, when the author illustrates an Example, the mathematics is usually down the center of the page, and if the students don't know what mathematical operation was performed, they will look to the right for some brief statement of help. That's not how we teach; we don't write out an Example on the board and then say, "Class, guess what I just did!" Instead we lead our students, telling them what step is coming and then performing that mathematical step *together*—and reading naturally from left to right. Student reviewers have said that the Examples in this book are easy to read; that's because *your* voice is right there with them, working through problems *together*.

EXAMPLE 1 Graphing a Quadratic Function Given in Standard Form

Graph the quadratic function $f(x) = (x - 3)^2 - 1$.

Solution:

Step 1 The parabola opens up.	$a = 1$, so $a > 0$
Step 2 Determine the vertex.	$(h, k) = (3, -1)$
Step 3 Find the y-intercept.	$f(0) = (-3)^2 - 1 = 8$

Skills and Concepts (Learning Objectives and Exercises)

In my experience as a mathematics teacher/instructor/professor, I find skills to be on the micro level and concepts on the macro level of understanding mathematics. I believe that too often skills are emphasized at the expense of conceptual understanding. I have purposely separated *learning objectives* at the beginning of every section into two categories: *skills objectives*—what students should be able to do; and *conceptual objectives*—what students should understand. At the beginning of every class I discuss the learning objectives for the day—both skills and concepts. These are reinforced with both skills exercises and conceptual exercises.

SECTION 2.3 LINES

SKILLS OBJECTIVES

- Determine x- and y-intercepts of a line.
- Calculate the slope of a line.
- Find the equation of a line using slope–intercept form.
- Find the equation of a line using point–slope form.
- Find the equation of a line that is parallel or perpendicular to a given line.

CONCEPTUAL OBJECTIVES

- Classify lines as rising, falling, horizontal, and vertical.
- Understand slope as a rate of change.
- Associate two lines having the same slope with the graph of parallel lines.
- Associate two lines having negative reciprocal slopes with the graph of perpendicular lines.

Catch the Mistake

Have you ever made a mistake (or had a student bring you his or her homework with a mistake) and you go over it and over it and can't find the mistake? It's often easier to simply take out a new sheet of paper and solve it from scratch again than it is to actually find the mistake. Finding the mistake demonstrates a higher level of understanding. I include a few *Catch the Mistake* exercises in each section that demonstrate a common mistake that I have seen in my experience. I use these in class (either as a whole or often in groups), which leads to student discussion and offers an opportunity for formative assessment in real time.

CATCH THE MISTAKE

In Exercises 95–98, explain the mistake that is made.

95. Solve the equation: $4e^x = 9$.

Solution:

Take the natural log of both sides. $\ln(4e^x) = \ln 9$

Apply the property of inverses. $4x = \ln 9$

Solve for x. $x = \frac{\ln 9}{4} \approx 0.55$

This is incorrect. What mistake was made?

96. Solve the equation: $\log(x) + \log(3) = 1$.

Solution:

97. Solve the equation: $\log(x) + \log(x + 3) = 1$ for x.

Solution:

Apply the product property (5). $\log(x^2 + 3x) = 1$

Exponentiate both sides (base 10). $10^{\log(x^2+3x)} = 10^1$

Apply the property of inverses. $x^2 + 3x = 10$

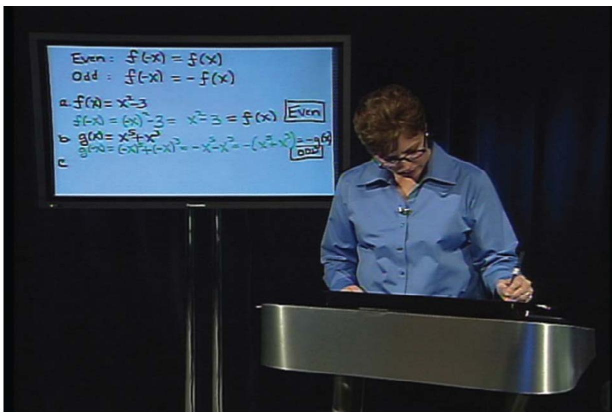
Factor. $(x + 5)(x - 2) = 0$

Solve for x. $x = -5$ and $x = 2$

This is incorrect. What mistake was made?

Lecture Videos by the Author

To ensure consistency in the students' learning experiences, I authored the videos myself. Throughout the book wherever a student sees the video icon, that indicates a video. These videos provide a mini lecture in that the chapter openers and chapter summaries are more like class discussion and selected Examples. Your Turns throughout the book also have an accompanying video of me working out that exact problem.



New to the Third Edition

The first edition was *my* book, the second edition was *our* book, and this third edition is *our even better* book. I've incorporated some specific line-by-line suggestions from reviewers throughout the exposition, added some new Examples, and added over 200 new Exercises. The three main global upgrades to the third edition are a new Chapter Map with Learning Objectives, End-of-chapter Inquiry-Based Learning Projects, and additional Applications Exercises in areas such as Business, Economics, Life Sciences, Health Sciences, and Medicine. A section (2.5*) on Linear Regression was added, as well as some technology exercises on Quadratic, Exponential, and Logarithmic Regression.

Learning Objectives

LEARNING OBJECTIVES

- Find the domain and range of a function.
- Sketch the graphs of common functions.
- Sketch graphs of general functions employing translations of common functions.
- Perform composition of functions.
- Find the inverse of a function.
- Model applications with functions using variation.

Inquiry-Based Learning Projects

CHAPTER 4 INQUIRY-BASED LEARNING PROJECT

Discovering the Connection between the Standard Form of a Quadratic Function and Transformations of the Square Function

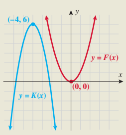
In Chapter 3, you saw that if you are familiar with the graphs of a small library of common functions, you can sketch the graphs of many related functions using transformation techniques. These ideas will help you here as you discover the relationship between the standard form of a quadratic function and its graph.

Let G and H be functions with:

$$G(x) = F(x - 1) + 3 \quad \text{and} \quad H(x) = -F(x + 2) - 4$$

where $F(x) = x^2$.

1. For this part, consider the function G.
   a. List the transformation you'd use to sketch the graph of G from the graph of F.
   b. Write an equation for $G(x)$ in the form $G(x) = a(x - h)^2 + k$. This is called the **standard form** of a quadratic function. What are the values of a, h, and k?
   c. The **vertex**, or turning point, of the graph of $F(x) = x^2$ is (0, 0). How can you use the transformations you listed in part (a) to determine the coordinates of the vertex of the graph of G?
   d. The vertical line that passes through the vertex of a parabola is called its **axis of symmetry**. The axis of symmetry of the graph of $F(x) = x^2$ is the y-axis, or the vertical line with equation $x = 0$. How can you determine the axis of symmetry of the graph of G? Write the equation of this line.
   e. Sketch graphs of F and G.
2. Next consider the function H given above.
   a. List the transformations that will produce the graph of H from the graph of F.
   b. Write an equation for $H(x)$ in standard form. What are the values of a, h, and k?
   c. What are the coordinates of the vertex of the graph of H? How do the transformations you listed in part (a) help you determine this?
   d. Determine the equation of the axis of symmetry of the graph of H.
   e. Sketch graphs of F and H.
3. a. What do you know about the graph of a quadratic function just by looking at its equation in standard form, $f(x) = a(x - h)^2 + k$?
   b. Shown below are the graphs of $F(x) = x^2$ and another quadratic function, $y = K(x)$. Write the equation of K in standard form. *Hint:* Think about the transformations.



466

Applications to Business, Economics, Health Sciences, and Medicine

APPLICATIONS

49. **Area.** Find the area enclosed by the system of inequalities. $y > |x|$, $y < 2$

50. **Area.** Find the area enclosed by the system of inequalities. $y < |x|$, $x \geq 0$, $y \geq 0$, $x < 3$

51. **Area.** Find the area enclosed by the system of linear inequalities (assume $y \geq 0$). $5x + y \leq 10$, $x \geq 0$, $x \leq 1$

52. **Area.** Find the area enclosed by the system of linear inequalities (assume $y \geq 0$). $-5x + y \leq 0$, $x \geq 1$, $x \leq 2$

53. **Hurricanes.** After back-to-back-to-back-to-back hurricanes (Charley, Frances, Ivan, and Jeanne) in Florida in the summer of 2004, FEMA sent disaster relief trucks to Florida. Floridians mainly needed drinking water and generators. Each truck could carry no more than 6000 pounds of cargo or 2400 cubic feet of cargo. Each case of bottled water takes up 1 cubic foot of space and weighs 25 pounds. Each generator takes up 20 cubic feet and weighs 150 pounds. Let x represent the number of cases of water and y represent the number of generators, and write a system of linear inequalities that describes the number of generators and cases of water each truck can haul to Florida.

54. **Hurricanes.** Repeat Exercise 53 with a smaller truck and different supplies. Suppose the smaller trucks that can haul 2000 pounds and 1500 cubic feet of cargo are used to haul plywood and tarps. A case of plywood is 60 cubic feet and weighs 500 pounds. A case of tarps is 10 cubic feet and weighs 50 pounds. Letting x represent the number of cases of plywood and y represent the number of cases of tarps, write a system of linear inequalities that describes the number of cases of tarps and plywood each truck can haul to Florida. Graph the system of linear inequalities.

55. **Health.** A diet must be designed to provide at least 275 units of calcium, 125 units of iron, and 200 units of Vitamin B. Each ounce of food A contains 10 units of calcium, 15 units of iron, and 20 units of vitamin B. Each ounce of food B contains 20 units of calcium, 10 units of iron, and 15 units of vitamin B.
    a. Find a system of inequalities to describe the different quantities of food that may be used (let x = the number of ounces of food A and y = the number of ounces of food B).
    b. Graph the system of inequalities.
    c. Using the graph found in part (b), find two possible solutions (there are infinitely many).

56. **Health.** A diet must be designed to provide at least 350 units of calcium, 175 units of iron, and 225 units of Vitamin B. Each ounce of food A contains 15 units of calcium, 25 units of iron, and 20 units of vitamin B. Each ounce of food B contains 25 units of calcium, 10 units of iron, and 10 units of vitamin B.
    a. Find a system of inequalities to describe the different quantities of food that may be used (let x = the number of ounces of food A and y = the number of ounces of food B).
    b. Graph the system of inequalities.
    c. Using the graph found in part (b), find two possible solutions (there are infinitely many).

57. **Business.** A manufacturer produces two types of computer mouse: USB wireless mouse and a Bluetooth mouse. Past sales indicate that it is necessary to produce at least twice as many USB wireless mice than Bluetooth mice. To meet demand, the manufacturer must produce at least 1000 computer mice per hour.
    a. Find a system of inequalities describing the production levels of computer mice. Let x be the production level for USB wireless mouse and y be the production level for Bluetooth mouse.
    b. Graph the system of inequalities describing the production levels of computer mice.
    c. Use your graph in part (b) to find two possible solutions.

58. **Business.** A manufacturer produces two types of mechanical pencil lead: 0.5 millimeter and 0.7 millimeter. Past sales indicate that it is necessary to produce at least 50% more 0.5 millimeter lead than 0.7 millimeter lead. To meet demand, the manufacturer must produce at least 10,000 pieces of pencil lead per hour.
    a. Find a system of inequalities describing the production levels of pencil lead. Let x be the production level for 0.5 millimeter pencil lead and y be the production level for 0.7 millimeter pencil lead.
    b. Graph the system of inequalities describing the production levels of pencil lead.
    c. Use your graph in part (b) to find two possible solutions.

FEATURE	BENEFIT TO STUDENT
Chapter Opening Vignette	Piques the student's interest with a real-world application of material presented in the chapter. Later in the chapter, the same concept from the vignette is reinforced.
Chapter Overview, Flowchart, and Learning Objectives	Students see the big picture of how topics relate and overarching learning objectives are presented.
Skills and Conceptual Objectives	Skills objectives represent what students should be able to do. Conceptual objectives emphasize a higher level global perspective of concepts.
Clear, Concise, and Inviting Writing Style, Tone, and Layout	Students are able to *read* this book, which reduces math anxiety and promotes student success.
Parallel Words and Math	Increases students' ability to read and understand examples with a seamless representation of their instructor's class (instructor's voice and what they would write on the board).
Common Mistakes	Addresses a different learning style: teaching by counter-example. Demonstrates common mistakes so that students understand why a step is incorrect and reinforces the correct mathematics.
Color for Pedagogical Reasons	Particularly helpful for visual learners when they see a function written in red and then its corresponding graph in red or a function written in blue and then its corresponding graph in blue.
Study Tips	Reinforces specific notes that you would want to emphasize in class.
Author Videos	Gives students a mini class of several examples worked by the author.
Your Turn	Engages students during class, builds student confidence, and assists instructor in real-time assessment.
Catch the Mistake Exercises	Encourages students to assume the role of teacher—demonstrating a higher mastery level.
Conceptual Exercises	Teaches students to think more globally about a topic.
Inquiry-Based Learning Project	Lets students *discover* a mathematical identify, formula, etc. that is derived in the book.
Modeling OUR World	Engages students in a modeling project of a timely subject: global climate change.
Chapter Review	Key ideas and formulas are presented section by section in a chart. Improves study skills.
Chapter Review Exercises	Improves study skills.
Chapter Practice Test	Offers self-assessment and improves study skills.
Cumulative Test	Improves retention.

Instructor Supplements

INSTRUCTOR'S SOLUTIONS MANUAL (ISBN: 978-1-118-13759-8)
- Contains worked-out solutions to all exercises in the text.

INSTRUCTOR'S MANUAL
Authored by Cynthia Young, the manual provides practical advice on teaching with the text, including:
- sample lesson plans and homework assignments
- suggestions for the effective utilization of additional resources and supplements
- sample syllabi
- Cynthia Young's Top 10 Teaching Tips & Tricks
- online component featuring the author presenting these Tips & Tricks

ANNOTATED INSTRUCTOR'S EDITION (ISBN: 978-1-118-13491-7)
- Displays answers to all exercise questions, which can be found in the back of the book.
- Provides additional classroom examples within the standard difficulty range of the in-text exercises, as well as challenge problems to assess your students' mastery of the material.

POWERPOINT SLIDES
- For each section of the book, a corresponding set of lecture notes and worked-out examples are presented as PowerPoint slides, available on the Book Companion Site (www.wiley.com/college/young) and *WileyPLUS*.

TEST BANK
- Contains approximately 900 questions and answers from every section of the text.

COMPUTERIZED TEST BANK
Electonically enhanced version of the Test Bank that
- contains approximately 900 algorithmically-generated questions.
- allows instructors to freely edit, randomize, and create questions.
- allows instructors to create and print different versions of a quiz or exam.
- recognizes symbolic notation.

BOOK COMPANION WEBSITE (WWW.WILEY.COM/COLLEGE/YOUNG)
- Contains all instructor supplements listed plus a selection of personal response system questions.

WILEYPLUS
- Features a full-service, digital learning environment, including additional resources for instructors, such as assignable homework exercises, tutorials, gradebook, and integrated links between the online version of the text and supplements.

Student Supplements

STUDENT SOLUTIONS MANUAL (ISBN: 978-1-118-13758-1)
- Includes worked-out solutions for all odd problems in the text.

BOOK COMPANION WEBSITE (WWW.WILEY.COM/COLLEGE/YOUNG)
- Provides additional resources for students to enhance the learning experience.

What Do Students Receive with *WileyPLUS*?

A Research-Based Design

WileyPLUS provides an online environment that integrates relevant resources, including the entire digital textbook, in an easy-to-navigate framework that helps students study more effectively.

- *WileyPLUS* adds structure by organizing textbook content into smaller, more manageable "chunks."
- Related media, examples, and sample practice items reinforce the learning objectives.
- Innovative features such as visual progress tracking, and self-evaluation tools improve time management and strengthen areas of weakness.

One-on-One Engagement

With *WileyPLUS*, students receive 24/7 access to resources that promote positive learning outcomes. Students engage with related examples (in various media) and sample practice items, including:

- Self-Study Quizzes
- Video Quizzes
- Proficiency Exams
- Guided Online (GO) Tutorial Problems
- Concept Questions
- Lecture Videos by Cynthia Young, including chapter introductions, chapter summaries, and selected video examples.

Measurable Outcomes

Throughout each study session, students can assess their progress and gain immediate feedback. *WileyPLUS* provides precise reporting of strengths and weaknesses, as well as individualized quizzes, so that students are confident they are spending their time on the right things. With *WileyPLUS*, students always know the exact outcome of their efforts.

What Do Instructors Receive with *WileyPLUS*?

WileyPLUS provides reliable, customizable resources that reinforce course goals inside and outside of the classroom, as well as visibility into individual student progress. Pre-created materials and activities help instructors optimize their time.

Customizable Course Plan

WileyPLUS comes with a pre-created Course Plan designed by a subject matter expert uniquely for this course.

Pre-Created Activity Types Include:

- Questions
- Readings and Resources
- Print Tests

Course Materials and Assessment Content

- Lecture Notes PowerPoint Slides
- Instructor's Manual
- Question Assignments (all end-of-chapter problems coded algorithmically with hints, links to text, whiteboard/show work feature, and instructor controlled problem solving help)

Gradebook

WileyPLUS provides instant access to reports on trends in class performance, student use of course materials, and progress toward learning objectives, helping inform decisions and drive classroom discussions.

Acknowledgments

I want to express my sincerest gratitude to the entire Wiley team. I've said this before, and I will say it again: Wiley is the right partner for me. There is a reason that my dog is named Wiley—she's smart, competitive, a team player, and most of all, a joy to be around. There are several people within Wiley to whom I feel the need to express my appreciation: first and foremost to Laurie Rosatone who convinced Wiley Higher Ed to invest in a young assistant professor's vision for a series and who has been unwavering in her commitment to student learning. To my editor Joanna Dingle whose judgment I trust in both editorial and preschool decisions; thank you for surpassing my greatest expectations for an editor. To the rest of the ladies on the math editorial team (Jen Brady, Beth Pearson, and Liz Baird), you are all first class! This revision was planned and executed exceptionally well thanks to you three. To the math marketing specialists Jonathan Cottrell and Jen Wreyford, thank you for helping reps tell my story: you both are outstanding at your jobs. To Kerry Weinstein, thank you for your attention to detail. To the art and illustration folks (Jennifer MacMillan, Sandra Rigby, and Dennis Ormond), thank you for bringing to life all of the sketches and figures. And finally, I'd like to thank all of the Wiley reps: thank you for your commitment to my series and your tremendous efforts to get professors to adopt this book for their students.

I would also like to thank all of the contributors who helped us make this *our even better book*. I'd first like to thank Mark McKibben. He is known as the author of the solutions manuals that accompany this series, but he is much more than that. Mark, thank you for making this series a priority, for being so responsive, and most of all for being my "go-to" person to think through ideas. I'd also like to especially thank Jodi B.A. McKibben who is a statistician and teamed with Mark to develop the new regression material. I'd like to thank Steve Davis who was the inspiration for the Inquiry-Based Learning Projects and a huge thanks to Lyn Riverstone who developed all of the IBLPs. Special thanks to Laura Watkins for finding applications that are real and timely and to Ricki Alexander for updating all of the Technology Tips. I'd also like to thank Becky Schantz for her environmental problems (I now use AusPens because of Becky).

I'd also like to thank the following reviewers whose input helped make this book even better.

Aaron Anderson, *Hillsborough Community College*
Bernadette Antkoviak, *Harrisburg Area Community College*
Jan Archibald, *Ventura College*
Shari Beck, *Navarro College*
Patricia K. Bezona, *Valdosta State University*
Connie Buller, *Metropolitan Community College*
James Carolan, *Wharton County Junior College*
Diane Cook, *Collegiate High School at Northwest Florida State College*
Doris C. Cowan, *Polk State College*
Jean Davis, *Texas State University*
Nerissa Felder, *Polk State College*
Sunshine Gibbons, *Southeast Missouri State University*
Mehran Hassanpour, *South Texas College*
Tom Hayes, *Montana State University – Bozeman*
Celeste Hernandez, *Richland College*
Carolyn Horseman, *Polk State College*
Dianne Marquart, *Valdosta State University*
Maria Luisa Mendez, *Laredo Community College*
Lily Rai, *South Texas College*
Leela Rakesh, *Central Michigan University*
Denise Reid, *Valdosta State University*
Linda Tansil, *Southeast Missouri State University*

And a special thanks to our student reviewer Luis Suarez del Rio.

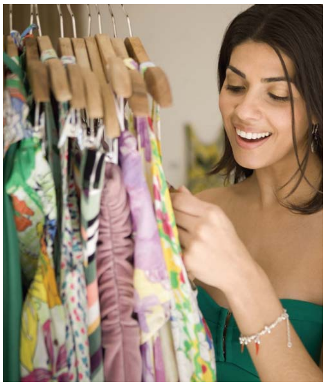
Table of Contents


Getty Images/Blend Images/Getty Images

Garry Wade/Taxi/Getty Images

Radius Images/©Corbis

Vetta/Getty Images



John Giustina/Superstock

Focus on Sport/Getty Images

Richard T. Nowitz/Photo Researchers, Inc.

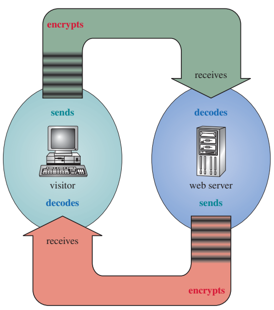
© Kathy Yturralde/Alamy

© Jason Brindel Commercial/Alamy

Q. Sakamaki/Redux Pictures

Q. Sakamaki/Redux Pictures

Paul J.Richards/AFP/Getty Images



Paul Souders/The Image Bank/Getty Images

A BASIC STRATEGY FOR BLACKJACK

Dealer's Up Card

Your Hand		2	3	4	5	6	7	8	9	10	A
I	17+	S	S	S	S	S	S	S	S	S	S
	16	S	S	S	S	S	H	H	H	H	H
	15	S	S	S	S	S	H	H	H	H	H
	14	S	S	S	S	S	H	H	H	H	H
	13	S	S	S	S	S	H	H	H	H	H
	12	H	H	S	S	S	H	H	H	H	H
II	11	D	D	D	D	D	D	D	D	D	H
	10	D	D	D	D	D	D	D	D	H	H
	9	H	D	D	D	D	H	H	H	H	H
	5 - 8	H	H	H	H	H	H	H	H	H	H
III	A, 8 - 10	S	S	S	S	S	S	S	S	S	S
	A, 7	S	D	D	D	D	S	S	H	H	H
	A, 6	H	D	D	D	D	H	H	H	H	H
	A, 5	H	H	D	D	D	H	H	H	H	H
	A, 4	H	H	D	D	D	H	H	H	H	H
	A, 3	H	H	H	D	D	H	H	H	H	H
	A, 2	H	H	H	D	D	H	H	H	H	H
IV	A, A; 8, 8	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
	10, 10	S	S	S	S	S	S	S	S	S	S
	9, 9	SP	SP	SP	SP	SP	S	SP	SP	S	S
	7, 7	SP	SP	SP	SP	SP	SP	H	H	H	H
	6, 6	H	SP	SP	SP	SP	H	H	H	H	H
	5, 5	D	D	D	D	D	D	D	D	H	H
	4, 4	H	H	H	H	H	H	H	H	H	H
	3, 3	H	H	SP	SP	SP	SP	H	H	H	H
	2, 2	H	H	SP	SP	SP	SP	H	H	H	H

When doubling down after splitting is allowed, split: 2's, 3's, 7's vs 2-7; 4's vs 5 or 6; 6's vs 2-6

When surrender is allowed, surrender 9, 7 or 10, 6 vs 9, 10, A; 9, 6 or 10, 5 vs 10

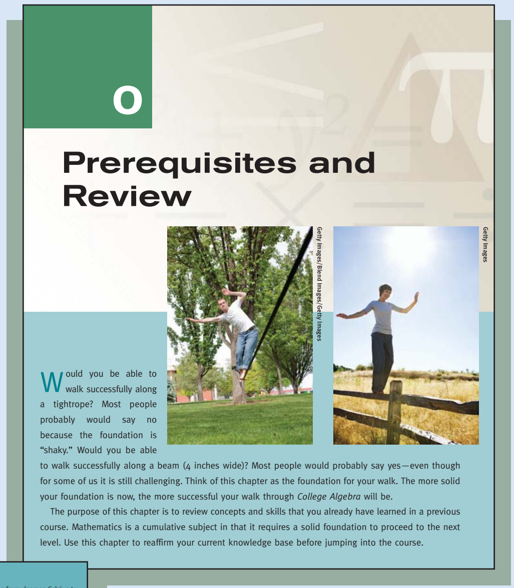
HIT STAND DOUBLE DOWN SPLIT

A Note from the Author to the Student

I wrote this text with careful attention to ways in which to make your learning experience more successful. If you take full advantage of the unique features and elements of this textbook, I believe your experience will be fulfilling and enjoyable. Let's walk through some of the special book features that will help you in your study of algebra and trigonometry.

Prerequisites and Review (Chapter 0)

A comprehensive review of prerequisite knowledge (intermediate algebra topics) in Chapter 0 provides a brush up on knowledge and skills necessary for success in the course.



0

Prerequisites and Review

Getty Images/Blend Images/Getty Images

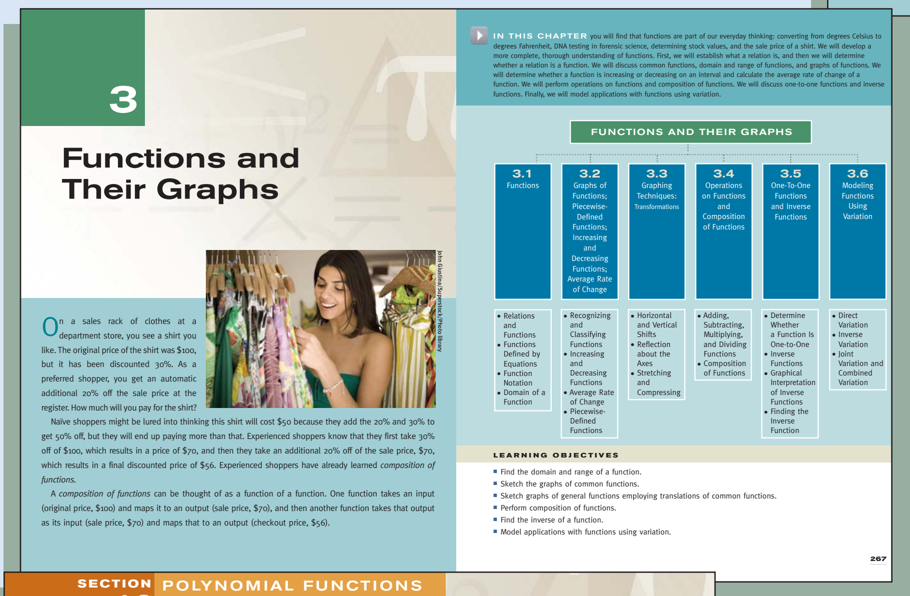
Getty Images

Would you be able to walk successfully along a tightrope? Most people probably would say no because the foundation is "shaky." Would you be able to walk successfully along a beam (4 inches wide)? Most people would probably say yes—even though for some of us it is still challenging. Think of this chapter as the foundation for your walk. The more solid your foundation is now, the more successful your walk through *College Algebra* will be.

The purpose of this chapter is to review concepts and skills that you already have learned in a previous course. Mathematics is a cumulative subject in that it requires a solid foundation to proceed to the next level. Use this chapter to reaffirm your current knowledge base before jumping into the course.

Clear, Concise, and Inviting Writing

Special attention has been made to present an engaging, clear, precise narrative in a layout that is easy to use and designed to reduce any math anxiety you may have.



3

Functions and Their Graphs

John Giustina/Superstock/Photo library

On a sales rack of clothes at a department store, you see a shirt you like. The original price of the shirt was $100, but it has been discounted 30%. As a preferred shopper, you get an automatic additional 20% off the sale price at the register. How much will you pay for the shirt?

Naïve shoppers might be lured into thinking this shirt will cost $50 because they add the 20% and 30% to get 50% off, but they will end up paying more than that. Experienced shoppers know that they first take 30% off of $100, which results in a price of $70, and then they take an additional 20% off of the sale price, $70, which results in a final discounted price of $56. Experienced shoppers have already learned *composition of functions.*

A *composition of functions* can be thought of as a function of a function. One function takes an input (original price, $100) and maps it to an output (sale price, $70), and then another function takes that output as its input (sale price, $70) and maps that to an output (checkout price, $56).

IN THIS CHAPTER you will find that functions are part of our everyday thinking: converting from degrees Celsius to degrees Fahrenheit, DNA testing in forensic science, determining stock values, and the sale price of a shirt. We will develop a more complete, thorough understanding of functions. First, we will establish what a relation is, and then we will determine whether a relation is a function. We will discuss common functions, domain and range of functions, and graphs of functions. We will determine whether a function is increasing or decreasing on an interval and calculate the average rate of change of a function. We will perform operations on functions and composition of functions. We will discuss one-to-one functions and inverse functions. Finally, we will model applications with functions using variation.

FUNCTIONS AND THEIR GRAPHS

3.1 Functions	3.2 Graphs of Functions; Piecewise-Defined Functions; Increasing and Decreasing Functions; Average Rate of Change	3.3 Graphing Techniques: Transformations	3.4 Operations on Functions and Composition of Functions	3.5 One-To-One Functions and Inverse Functions	3.6 Modeling Functions Using Variation
• Relations and Functions • Functions Defined by Equations • Function Notation • Domain of a Function	• Recognizing and Classifying Functions • Increasing and Decreasing Functions • Average Rate of Change • Piecewise-Defined Functions	• Horizontal and Vertical Shifts • Reflection about the Axes • Stretching and Compressing	• Adding, Subtracting, Multiplying, and Dividing Functions • Composition of Functions	• Determine Whether a Function Is One-to-One • Inverse Functions • Graphical Interpretation of Inverse Functions • Finding the Inverse Function	• Direct Variation • Inverse Variation • Joint Variation and Combined Variation

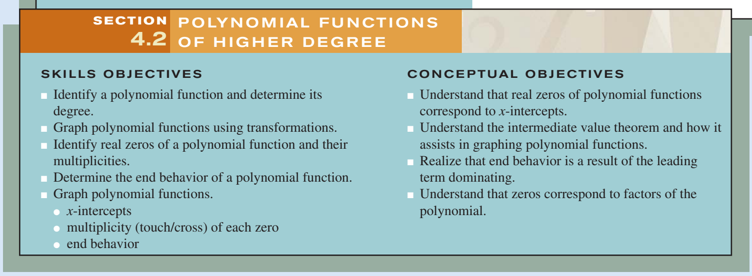
LEARNING OBJECTIVES

- Find the domain and range of a function.
- Sketch the graphs of common functions.
- Sketch graphs of general functions employing translations of common functions.
- Perform composition of functions.
- Find the inverse of a function.
- Model applications with functions using variation.

267

Chapter Introduction, Flow Chart, Section Headings, and Objectives

An opening vignette, flow chart, list of chapter sections, and chapter learning objectives give you an overview of the chapter.



SECTION 4.2 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE

SKILLS OBJECTIVES

- Identify a polynomial function and determine its degree.
- Graph polynomial functions using transformations.
- Identify real zeros of a polynomial function and their multiplicities.
- Determine the end behavior of a polynomial function.
- Graph polynomial functions.
  - x-intercepts
  - multiplicity (touch/cross) of each zero
  - end behavior

CONCEPTUAL OBJECTIVES

- Understand that real zeros of polynomial functions correspond to x-intercepts.
- Understand the intermediate value theorem and how it assists in graphing polynomial functions.
- Realize that end behavior is a result of the leading term dominating.
- Understand that zeros correspond to factors of the polynomial.

Skills and Conceptual Objectives

For every section, objectives are further divided by skills *and* concepts so you can see the difference between solving problems and truly understanding concepts.

Examples

Examples pose a specific problem using concepts already presented and then work through the solution. These serve to enhance your understanding of the subject matter.

EXAMPLE 9 Evaluating the Difference Quotient

For the function $f(x) = x^2 - x$, find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$.

Solution:

Use placeholder notation for the function $f(x) = x^2 - x$. $\quad f(\square) = (\square)^2 - (\square)$

Calculate $f(x + h)$. $\quad f(x+h) = (x+h)^2 - (x+h)$

Write the difference quotient. $\quad \frac{f(x+h) - f(x)}{h}$

Let $f(x + h) = (x + h)^2 - (x + h)$ and $f(x) = x^2 - x$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{[(x+h)^2 - (x+h)]}^{f(x+h)} - \overbrace{[x^2 - x]}^{f(x)}}{h} \qquad h \neq 0$$

Eliminate the parentheses inside the first set of brackets. $\quad = \frac{[x^2 + 2xh + h^2 - x - h] - [x^2 - x]}{h}$

Eliminate the brackets in the numerator. $\quad = \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$

Combine like terms. $\quad = \frac{2xh + h^2 - h}{h}$

Factor the numerator. $\quad = \frac{h(2x + h - 1)}{h}$

Divide out the common factor, h. $\quad = \boxed{2x + h - 1} \qquad h \neq 0$

YOUR TURN Evaluate the difference quotient for $f(x) = x^2 - 1$.

Your Turn

Immediately following many examples, you are given a similar problem to reinforce and check your understanding. This helps build confidence as you progress in the chapter. These are ideal for in-class activity or for preparing for homework later. Answers are provided in the margin for a quick check of your work.

COMMON MISTAKE

A common misunderstanding is to interpret the notation $f(x + 1)$ as a sum: $f(x + 1) \neq f(x) + f(1)$.

CORRECT

Write the original function.

$f(x) = x^2 - 3x$

Replace the argument x with a placeholder.

$f(\square) = (\square)^2 - 3(\square)$

Substitute $x + 1$ for the argument.

$f(x + 1) = (x + 1)^2 - 3(x + 1)$

Eliminate the parentheses.

$f(x + 1) = x^2 + 2x + 1 - 3x - 3$

Combine like terms.

$\boxed{f(x + 1) = x^2 - x - 2}$

INCORRECT

The **ERROR** is in interpreting the notation as a sum.

$f(x + 1) \neq f(x) + f(1)$

$\neq x^2 - 3x - 2$

Common Mistake/ Correct vs. Incorrect

In addition to standard examples, some problems are worked out both correctly and incorrectly to highlight common errors students make. Counter examples like these are often an effective learning approach for many students.

Parallel Words and Math

This text reverses the common textbook presentation of examples by placing the explanation in words *on the left* and the mathematics in parallel *on the right*. This makes it easier for students to read through examples as the material flows more naturally from left to right and as commonly presented in class.

Words	**Math**
Write the interest formula for compounding continuously.	$A = Pe^{rt}$
Let $A = 2P$ (investment doubles).	$2P = Pe^{rt}$
Divide both sides of the equation by P.	$2 = e^{rt}$
Take the natural log of both sides of the equation.	$\ln 2 = \ln e^{rt}$
Simplify the right side by applying the property $\ln e^x = x$.	$\ln 2 = rt$
Divide both sides by r.	$t = \frac{\ln 2}{r}$
Approximate $\ln 2 \approx 0.7$.	$t \approx \frac{0.7}{r}$

Study Tips and Caution Notes

These marginal reminders call out important hints or warnings to be aware of related to the topic or problem.

Study Tip

The largest number of zeros a polynomial can have is equal to the degree of the polynomial.

CAUTION

$f \circ g \neq f \cdot g$

Technology Tips

These marginal notes provide problem solving instructions and visual examples using graphing calculators.

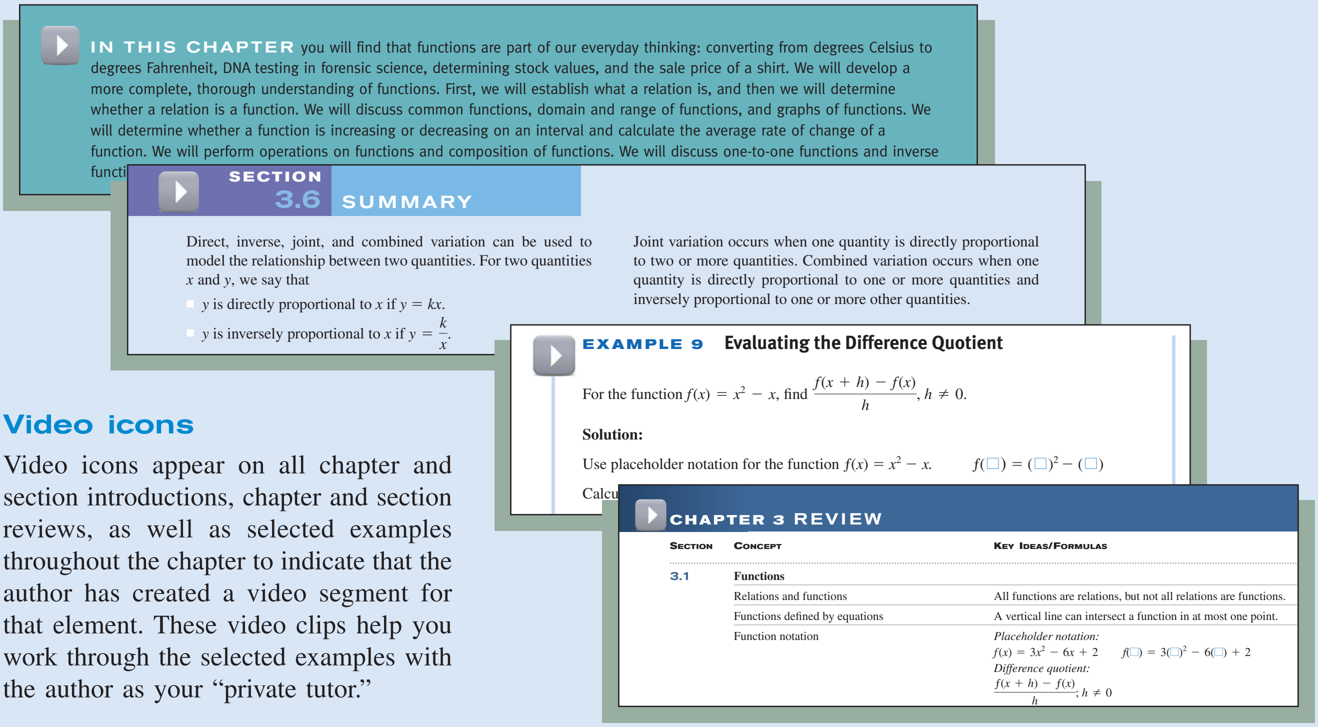
Technology Tip

A graphing utility can be used to evaluate $P(2)$. Enter $P(x) = 4x^5 - 3x^4 + 2x^3 - 7x^2 + 9x - 5$ as Y_1.

```
Plot1 Plot2 Plot3
\Y1=4X^5-3X^4+2X
^3-7X²+9X-5
\Y2=
```

To evaluate $P(2)$, press VARS ▶ Y-VARS 1:Function 1:Y_1 (2) ENTER

```
Y1(2)
                81
```


IN THIS CHAPTER you will find that functions are part of our everyday thinking: converting from degrees Celsius to degrees Fahrenheit, DNA testing in forensic science, determining stock values, and the sale price of a shirt. We will develop a more complete, thorough understanding of functions. First, we will establish what a relation is, and then we will determine whether a relation is a function. We will discuss common functions, domain and range of functions, and graphs of functions. We will determine whether a function is increasing or decreasing on an interval and calculate the average rate of change of a function. We will perform operations on functions and composition of functions. We will discuss one-to-one functions and inverse functi

SECTION 3.6 SUMMARY

Direct, inverse, joint, and combined variation can be used to model the relationship between two quantities. For two quantities x and y, we say that

- y is directly proportional to x if $y = kx$.
- y is inversely proportional to x if $y = \frac{k}{x}$.

Joint variation occurs when one quantity is directly proportional to two or more quantities. Combined variation occurs when one quantity is directly proportional to one or more quantities and inversely proportional to one or more other quantities.

EXAMPLE 9 Evaluating the Difference Quotient

For the function $f(x) = x^2 - x$, find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$.

Solution:

Use placeholder notation for the function $f(x) = x^2 - x$. $f(\square) = (\square)^2 - (\square)$

Calcu

CHAPTER 3 REVIEW

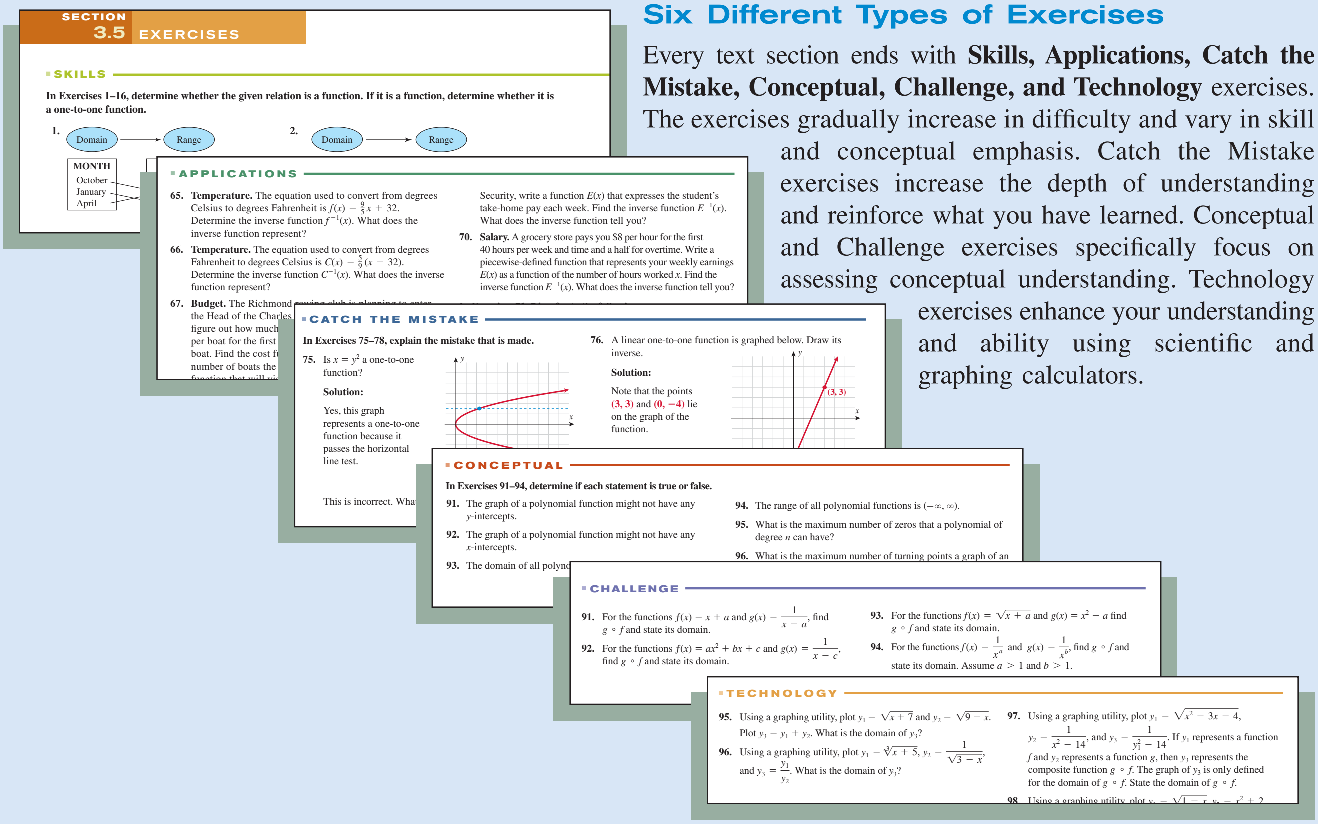
SECTION	CONCEPT	KEY IDEAS/FORMULAS
3.1	Functions	
	Relations and functions	All functions are relations, but not all relations are functions.
	Functions defined by equations	A vertical line can intersect a function in at most one point.
	Function notation	*Placeholder notation:* $f(x) = 3x^2 - 6x + 2$ $f(\square) = 3(\square)^2 - 6(\square) + 2$ *Difference quotient:* $\frac{f(x+h) - f(x)}{h}$; $h \neq 0$

Video icons

Video icons appear on all chapter and section introductions, chapter and section reviews, as well as selected examples throughout the chapter to indicate that the author has created a video segment for that element. These video clips help you work through the selected examples with the author as your "private tutor."

Six Different Types of Exercises

Every text section ends with **Skills, Applications, Catch the Mistake, Conceptual, Challenge, and Technology** exercises. The exercises gradually increase in difficulty and vary in skill and conceptual emphasis. Catch the Mistake exercises increase the depth of understanding and reinforce what you have learned. Conceptual and Challenge exercises specifically focus on assessing conceptual understanding. Technology exercises enhance your understanding and ability using scientific and graphing calculators.


SECTION 3.5 EXERCISES

SKILLS

In Exercises 1–16, determine whether the given relation is a function. If it is a function, determine whether it is a one-to-one function.

1. Domain → Range (MONTH: October, January, April)
2. Domain → Range

APPLICATIONS

65. **Temperature.** The equation used to convert from degrees Celsius to degrees Fahrenheit is $f(x) = \frac{9}{5}x + 32$. Determine the inverse function $f^{-1}(x)$. What does the inverse function represent?

66. **Temperature.** The equation used to convert from degrees Fahrenheit to degrees Celsius is $C(x) = \frac{5}{9}(x - 32)$. Determine the inverse function $C^{-1}(x)$. What does the inverse function represent?

67. **Budget.** The Richmond rowing club is planning to enter the Head of the Charles ... figure out how much ... per boat for the first ... boat. Find the cost f... number of boats the ...

Security, write a function $E(x)$ that expresses the student's take-home pay each week. Find the inverse function $E^{-1}(x)$. What does the inverse function tell you?

70. **Salary.** A grocery store pays you \$8 per hour for the first 40 hours per week and time and a half for overtime. Write a piecewise-defined function that represents your weekly earnings $E(x)$ as a function of the number of hours worked x. Find the inverse function $E^{-1}(x)$. What does the inverse function tell you?

CATCH THE MISTAKE

In Exercises 75–78, explain the mistake that is made.

75. Is $x = y^2$ a one-to-one function?

Solution:

Yes, this graph represents a one-to-one function because it passes the horizontal line test.

This is incorrect. Wha

76. A linear one-to-one function is graphed below. Draw its inverse.

Solution:

Note that the points (3, 3) and (0, −4) lie on the graph of the function.

CONCEPTUAL

In Exercises 91–94, determine if each statement is true or false.

91. The graph of a polynomial function might not have any y-intercepts.

92. The graph of a polynomial function might not have any x-intercepts.

93. The domain of all polyno

94. The range of all polynomial functions is $(-\infty, \infty)$.

95. What is the maximum number of zeros that a polynomial of degree n can have?

96. What is the maximum number of turning points a graph of an

CHALLENGE

91. For the functions $f(x) = x + a$ and $g(x) = \frac{1}{x - a}$, find $g \circ f$ and state its domain.

92. For the functions $f(x) = ax^2 + bx + c$ and $g(x) = \frac{1}{x - c}$, find $g \circ f$ and state its domain.

93. For the functions $f(x) = \sqrt{x + a}$ and $g(x) = x^2 - a$ find $g \circ f$ and state its domain.

94. For the functions $f(x) = \frac{1}{x^a}$ and $g(x) = \frac{1}{x^b}$, find $g \circ f$ and state its domain. Assume $a > 1$ and $b > 1$.

TECHNOLOGY

95. Using a graphing utility, plot $y_1 = \sqrt{x + 7}$ and $y_2 = \sqrt{9 - x}$. Plot $y_3 = y_1 + y_2$. What is the domain of y_3?

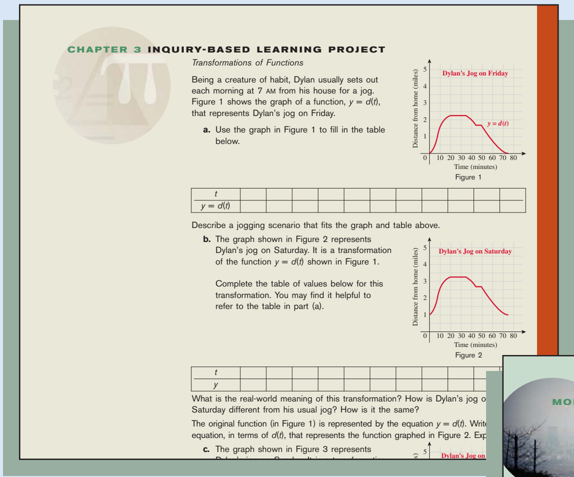
96. Using a graphing utility, plot $y_1 = \sqrt[3]{x + 5}$, $y_2 = \frac{1}{\sqrt{3 - x}}$ and $y_3 = \frac{y_1}{y_2}$. What is the domain of y_3?

97. Using a graphing utility, plot $y_1 = \sqrt{x^2 - 3x - 4}$, $y_2 = \frac{1}{x^2 - 14}$, and $y_3 = \frac{1}{y_1^2 - 14}$. If y_1 represents a function f and y_2 represents a function g, then y_3 represents the composite function $g \circ f$. The graph of y_3 is only defined for the domain of $g \circ f$. State the domain of $g \circ f$.

98. Using a graphing utility, plot $y_1 = \sqrt{1 - x}$, $y_2 = x^2 + 2$

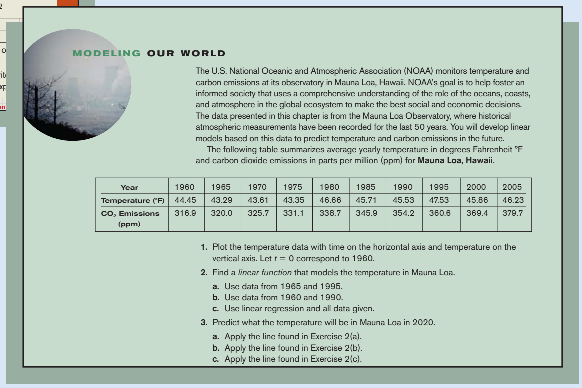
Inquiry-Based Learning Projects

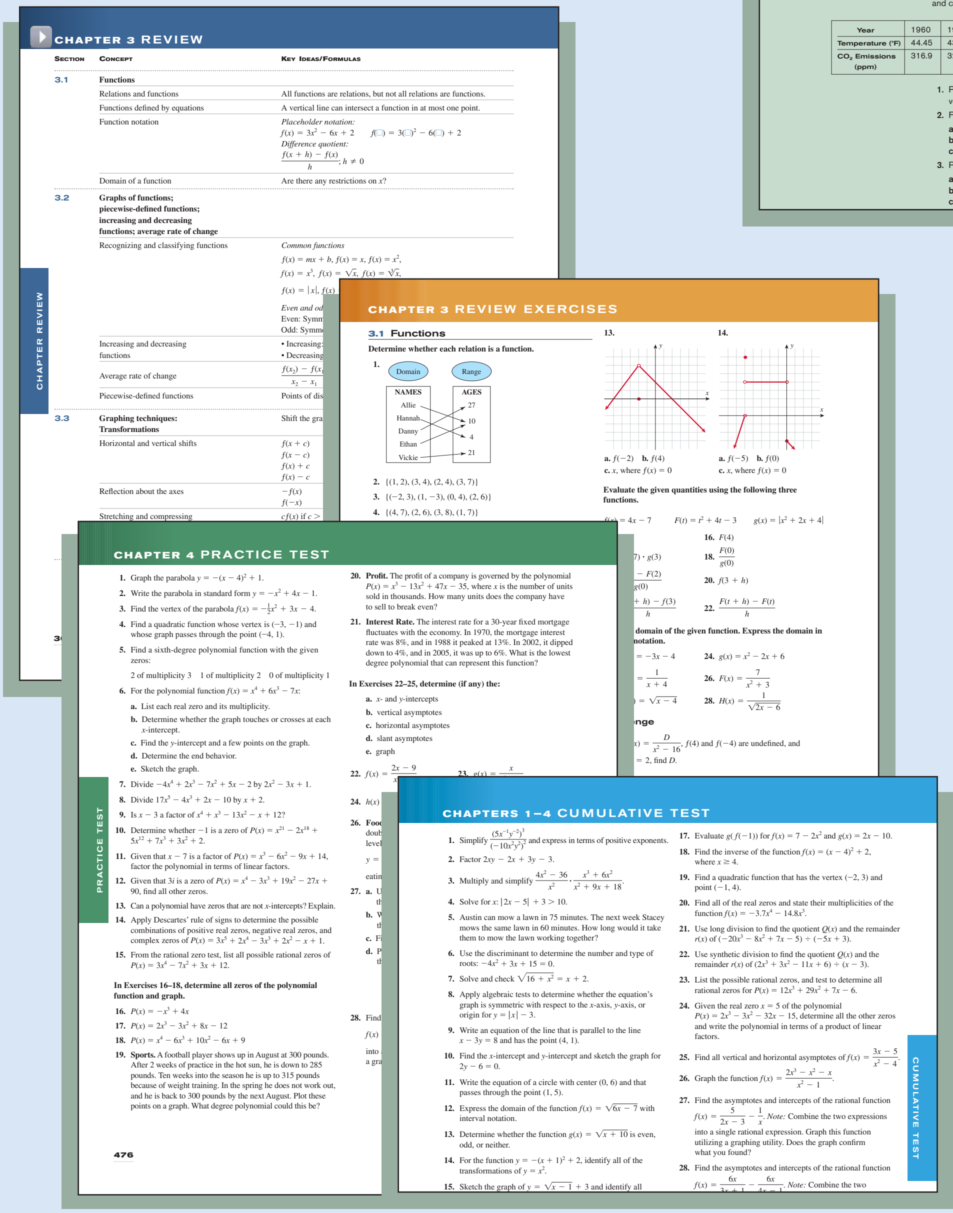
These end of chapter projects enable you to discover mathematical concepts on your own!



Modeling Our World

These unique end-of-chapter exercises provide a fun and interesting way to take what you have learned and model a real world problem. By using climate change as the continuous theme, these exercises can help you to develop more advanced modeling skills with each chapter while seeing how modeling can help you better understand the world around you.





Chapter Review, Review Exercises, Practice Test, Cumulative Test

At the end of every chapter, a summary review chart organizes the key learning concepts in an easy to use one or two-page layout. This feature includes key ideas and formulas, as well as indicating relevant pages and review exercises so that you can quickly summarize a chapter and study smarter. Review Exercises, arranged by section heading, are provided for extra study and practice. A Practice Test, without section headings, offers even more self-practice before moving on. A new Cumulative Test feature offers study questions based on all previous chapters' content, thus helping you build upon previously learned concepts.

Algebra and Trigonometry

Third Edition

Definitions, Rules, Formulas, and Graphs

Arithmetic Operations

$ab + ac = a(b + c)$

$\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

$\dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$\dfrac{\left(\dfrac{a}{b}\right)}{\left(\dfrac{c}{d}\right)} = \dfrac{ad}{bc}$

$a\left(\dfrac{b}{c}\right) = \dfrac{ab}{c}$

$\dfrac{a - b}{c - d} = \dfrac{b - a}{d - c}$

$\dfrac{ab + ac}{a} = b + c, \; a \neq 0$

$\dfrac{\left(\dfrac{a}{b}\right)}{c} = \dfrac{a}{bc}$

$\dfrac{a}{\left(\dfrac{b}{c}\right)} = \dfrac{ac}{b}$

Exponents and Radicals

$a^0 = 1, \; a \neq 0$

$\dfrac{a^x}{a^y} = a^{x-y}$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

$\sqrt[n]{a^m} = a^{m/n} = \left(\sqrt[n]{a}\right)^m$

$a^{-x} = \dfrac{1}{a^x}$

$(a^x)^y = a^{xy}$

$\sqrt{a} = a^{1/2}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{a} = a^{1/n}$

$\sqrt[n]{\left(\dfrac{a}{b}\right)} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$a^x a^y = a^{x+y}$

$(ab)^x = a^x b^x$

Absolute Value

1. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x > 0 \end{cases}$
2. If $|x| = c$, then $x = c$ or $x = -c$. $(c > 0)$
3. If $|x| < c$, then $-c < x < c$. $(c > 0)$
4. If $|x| > c$, then $x < -c$ or $x > c$. $(c > 0)$

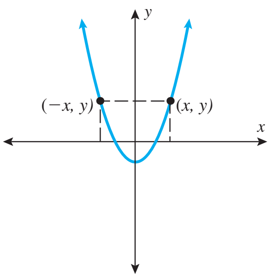
Properties of Logarithms

1. $\log_b(MN) = \log_b M + \log_b N$
2. $\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$
3. $\log_b M^p = p\log_b M$
4. $\log_b M = \dfrac{\log_a M}{\log_a b} = \dfrac{\ln M}{\ln b} = \dfrac{\log M}{\log b}$
5. $\log_b b^x = x$; $\ln e^x = x$
6. $b^{\log_b x} = x$; $e^{\ln x} = x \quad x > 0$

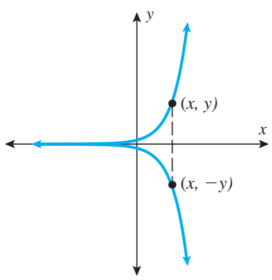
Special Factorizations

1. *Difference of two squares:*
   $A^2 - B^2 = (A + B)(A - B)$
2. *Perfect square trinomials:*
   $A^2 + 2AB + B^2 = (A + B)^2$
   $A^2 - 2AB + B^2 = (A - B)^2$
3. *Sum of two cubes:*
   $A^3 + B^3 = (A + B)\left(A^2 - AB + B^2\right)$
4. *Difference of two cubes:*
   $A^3 - B^3 = (A - B)\left(A^2 + AB + B^2\right)$

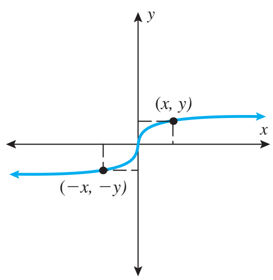
Symmetry



***y*-Axis Symmetry**



***x*-Axis Symmetry**



Origin Symmetry

FORMULAS/EQUATIONS

Distance Formula The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Midpoint Formula The midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Standard Equation of a Circle The standard equation of a circle of radius r with center at (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2$$

Slope Formula The slope m of the line containing the points (x_1, y_1) and (x_2, y_2) is

$$\text{slope } (m) = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2)$$

m is undefined if $x_1 = x_2$

Slope-Intercept Equation of a Line The equation of a line with slope m and y-intercept $(0, b)$ is $y = mx + b$

Point-Slope Equation of a Line The equation of a line with slope m containing the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

Quadratic Formula The solutions of the equation $ax^2 + bx + c = 0$, $a \neq 0$, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

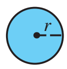
If $b^2 - 4ac > 0$, there are two distinct real solutions.

If $b^2 - 4ac = 0$, there is a repeated real solution.

If $b^2 - 4ac < 0$, there are two complex solutions (complex conjugates).

GEOMETRY FORMULAS

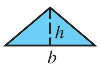
Circle



r = Radius, A = Area, C = Circumference

$A = \pi r^2 \quad C = 2\pi r$

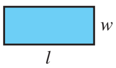
Triangle



b = Base, h = Height (Altitude), A = area

$A = \frac{1}{2}bh$

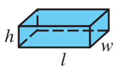
Rectangle



l = Length, w = Width, A = area, P = perimeter

$A = lw \quad P = 2l + 2w$

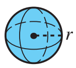
Rectangular Box



l = Length, w = Width, h = Height, V = Volume, S = Surface area

$V = lwh \quad S = 2lw + 2lh + 2wh$

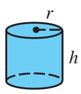
Sphere



r = Radius, V = Volume, S = Surface area

$V = \frac{4}{3}\pi r^3 \quad S = 4\pi r^2$

Right Circular Cylinder



r = Radius, h = Height, V = Volume, S = Surface area

$V = \pi r^2 h \quad S = 2\pi r^2 + 2\pi rh$

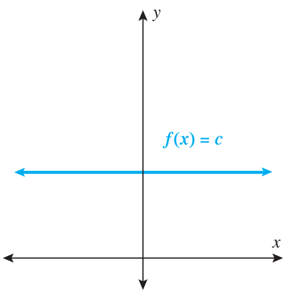
CONVERSION TABLE

1 centimeter	≈ 0.394 inch	1 joule	≈ 0.738 foot-pound	1 mile	≈ 1.609 kilometers
1 meter	≈ 39.370 inches	1 gram	≈ 0.035 ounce	1 gallon	≈ 3.785 liters
	≈ 3.281 feet	1 kilogram	≈ 2.205 pounds	1 pound	≈ 4.448 newtons
1 kilometer	≈ 0.621 mile	1 inch	≈ 2.540 centimeters	1 foot-lb	≈ 1.356 Joules
1 liter	≈ 0.264 gallon	1 foot	≈ 30.480 centimeters	1 ounce	≈ 28.350 grams
1 newton	≈ 0.225 pound		≈ 0.305 meter	1 pound	≈ 0.454 kilogram

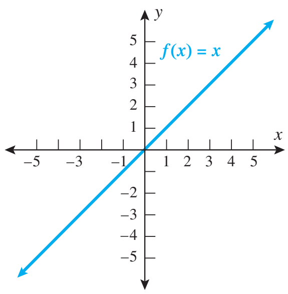
Functions

Constant Function	$f(x) = b$
Linear Function	$f(x) = mx + b$, where m is the slope and b is the y-intercept
Quadratic Function	$f(x) = ax^2 + bx + c,\ a \neq 0$ or $f(x) = a(x - h)^2 + k$ parabola vertex (h, k)
Polynomial Function	$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$
Rational Function	$R(x) = \dfrac{n(x)}{d(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + a_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$
Exponential Function	$f(x) = b^x, b > 0,\ b \neq 1$
Logarithmic Function	$f(x) = \log_b x, b > 0, b \neq 1$

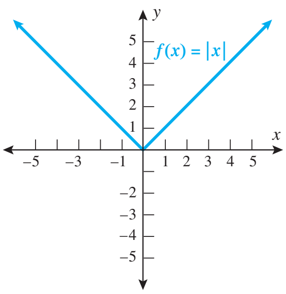
Graphs of Common Functions



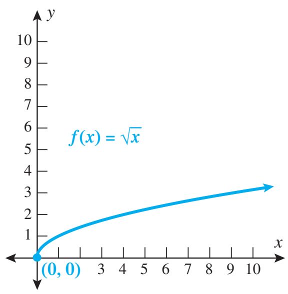
Constant Function



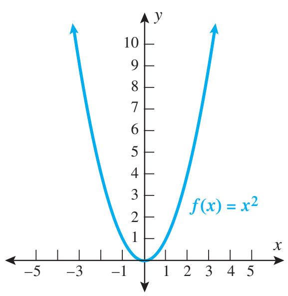
Identity Function



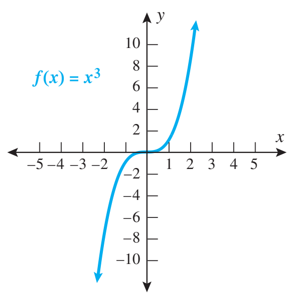
Absolute Value Function



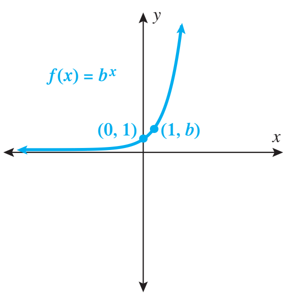
Square Root Function



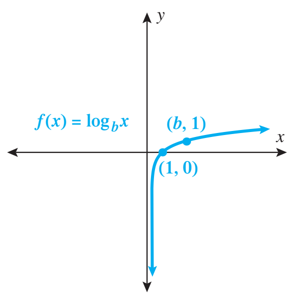
Square Function



Cube Function



Exponential Function



Logarithmic Function

Transformations

In each case, c represents a positive real number.

Function		**Draw the graph of f and:**
Vertical translations	$y = f(x) + c$	Shift f upward c units.
	$y = f(x) - c$	Shift f downward c units.
Horizontal translations	$y = f(x - c)$	Shift f to the right c units.
	$y = f(x + c)$	Shift f to the left c units.
Reflections	$y = -f(x)$	Reflect f about the x-axis.
	$y = f(-x)$	Reflect f about the y-axis.

Heron's Formula for Area

If the semiperimeter, s, of a triangle is

$$s = \frac{a + b + c}{2}$$

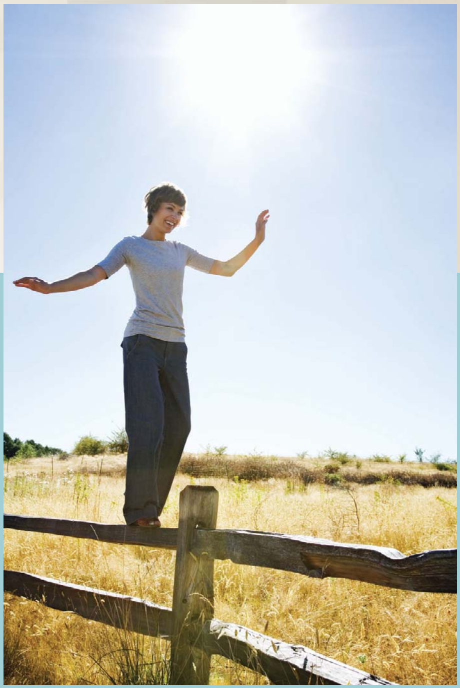
then the area of that triangle is

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

0

Prerequisites and Review

Getty Images/Blend Images/Getty Images



Garry Wade/Taxi/Getty Images

Would you be able to walk successfully along a tightrope? Most people probably would say no because the foundation is "shaky." Would you be able to walk successfully along a beam (4 inches wide)? Most people would probably say yes—even though for some of us it is still challenging. Think of this chapter as the foundation for your walk. The more solid your foundation is now, the more successful your walk through *College Algebra* will be.

The purpose of this chapter is to review concepts and skills that you already have learned in a previous course. Mathematics is a cumulative subject in that it requires a solid foundation to proceed to the next level. Use this chapter to reaffirm your current knowledge base before jumping into the course.

IN THIS CHAPTER real numbers, integer exponents, and scientific notation will be discussed, followed by rational exponents and radicals. Simplification of radicals and rationalization of denominators will be reviewed. Basic operations such as addition, subtraction, and multiplication of polynomials will be discussed followed by a review of how to factor polynomials. Rational expressions will be discussed and a brief overview of solving simple algebraic equations will be given. After reviewing all of these aspects of real numbers, this chapter will conclude with a review of complex numbers.

PREREQUISITES AND REVIEW

0.1 Real Numbers

- The Set of Real Numbers
- Approximations: Rounding and Truncation
- Order of Operations
- Properties of Real Numbers

0.2 Integer Exponents and Scientific Notation

- Integer Exponents
- Scientific Notation

0.3 Polynomials: Basic Operations

- Adding and Subtracting Polynomials
- Multiplying Polynomials
- Special Products

0.4 Factoring Polynomials

- Greatest Common Factor
- Factoring Formulas: Special Polynomial Forms
- Factoring a Trinomial as a Product of Two Binomials
- Factoring by Grouping
- A Strategy for Factoring Polynomials

0.5 Rational Expressions

- Rational Expressions and Domain Restrictions
- Simplifying Rational Expressions
- Multiplying and Dividing Rational Expressions
- Adding and Subtracting Rational Expressions
- Complex Rational Expressions

0.6 Rational Exponents and Radicals

- Square Roots
- Other (nth) Roots
- Rational Exponents

0.7 Complex Numbers

- The Imaginary Unit, i
- Adding and Subtracting Complex Numbers
- Multiplying Complex Numbers
- Dividing Complex Numbers
- Raising Complex Numbers to Integer Powers

LEARNING OBJECTIVES

- Understand that rational and irrational numbers together constitute the real numbers.
- Apply properties of exponents.
- Perform operations on polynomials.
- Factor polynomials.
- Simplify expressions that contain rational exponents.
- Simplify radicals.
- Write complex numbers in standard form.

SECTION 0.1 REAL NUMBERS

SKILLS OBJECTIVES

- Classify real numbers as rational or irrational.
- Round or truncate real numbers.
- Simplify expressions using correct order of operations.
- Evaluate algebraic expressions.
- Apply properties of real numbers.

CONCEPTUAL OBJECTIVES

- Understand that rational and irrational numbers are mutually exclusive and complementary subsets of real numbers.
- Learn the order of operations for real numbers.

The Set of Real Numbers

A **set** is a group or collection of objects that are called **members** or **elements** of the set. If *every* member of set B is also a member of set A, then we say B is a **subset** of A and denote it as $B \subset A$.

For example, the starting lineup on a baseball team is a subset of the entire team. The set of **natural numbers**, $\{1, 2, 3, 4, \ldots\}$, is a subset of the set of **whole numbers**, $\{0, 1, 2, 3, 4, \ldots\}$, which is a subset of the set of **integers**, $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, which is a subset of the set of *rational numbers*, which is a subset of the set of *real numbers*. The three dots, called an **ellipsis**, indicate that the pattern continues indefinitely.

If a set has no elements, it is called the **empty set**, or **null set**, and is denoted by the symbol $\varnothing$. The **set of real numbers** consists of two main subsets: *rational* and *irrational* numbers.

DEFINITION **Rational Number**

A **rational number** is a number that can be expressed as a quotient (ratio) of two integers, $\frac{a}{b}$, where the integer a is called the **numerator** and the integer b is called the **denominator** and where $b \neq 0$.

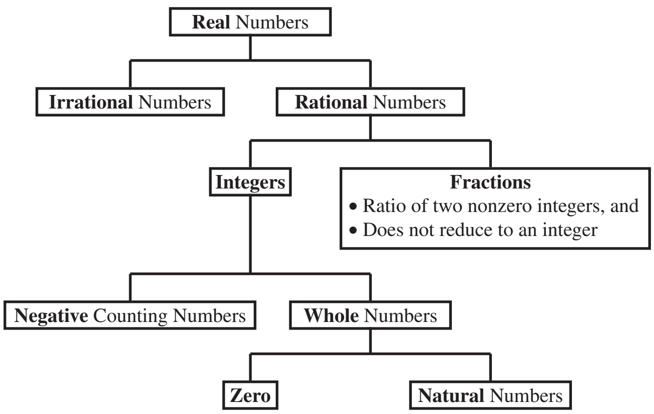
Rational numbers include all integers or all fractions that are ratios of integers. Note that any integer can be written as a ratio whose denominator is equal to 1. In decimal form, the rational numbers are those that terminate or are nonterminating with a repeated decimal pattern, which is represented with an overbar. Those decimals that do not repeat and do not terminate are **irrational numbers**. The numbers

$$5, \quad -17, \quad \frac{1}{3}, \quad \sqrt{2}, \quad \pi, \quad 1.37, \quad 0, \quad -\frac{19}{17}, \quad 3.66\overline{6}, \quad 3.2179\ldots$$

are examples of **real** numbers, where 5, -17, $\frac{1}{3}$, 1.37, 0, $-\frac{19}{17}$, and $3.66\overline{6}$ are rational numbers, and $\sqrt{2}$, π, and $3.2179\ldots$ are irrational numbers. It is important to note that the ellipsis following the last decimal digit denotes continuing in an irregular fashion, whereas the absence of such dots to the right of the last decimal digit implies the decimal expansion terminates.

Rational Number (Fraction)	Calculator Display	Decimal Representation	Description
$\frac{7}{2}$	3.5	3.5	Terminates
$\frac{15}{12}$	1.25	1.25	Terminates
$\frac{2}{3}$	0.666666666	$0.\overline{6}$	Repeats
$\frac{1}{11}$	0.09090909	$0.\overline{09}$	Repeats

Notice that the overbar covers the entire repeating pattern. The following figure and table illustrate the subset relationship and examples of different types of real numbers.

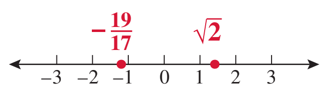


Study Tip

Every real number is either a rational number *or* an irrational number.

Symbol	Name	Description	Examples
$\mathbb{N}$	Natural numbers	Counting numbers	1, 2, 3, 4, 5, . . .
$\mathbb{W}$	Whole numbers	Natural numbers and zero	0, 1, 2, 3, 4, 5, . . .
$\mathbb{Z}$	Integers	Whole numbers and negative natural numbers	. . . , −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, . . .
$\mathbb{Q}$	Rational numbers	Ratios of integers: $\frac{a}{b}$ $(b \neq 0)$ • Decimal representation terminates, or • Decimal representation repeats	$-17, -\frac{19}{7}, 0, \frac{1}{3}, 1.37, 3.66\overline{6}, 5$
$\mathbb{I}$	Irrational numbers	Numbers whose decimal representation does *not* terminate or repeat	$\sqrt{2}$, 1.2179. . . , π
$\mathbb{R}$	Real numbers	Rational and irrational numbers	$\pi, 5, -\frac{2}{3}, 17.25, \sqrt{7}$

Since the set of real numbers can be formed by combining the set of rational numbers and the set of irrational numbers, then every real number is either rational or irrational. The set of rational numbers and the set of irrational numbers are both mutually exclusive (no shared elements) and complementary sets. The **real number line** is a graph used to represent the set of all real numbers.



EXAMPLE 1 **Classifying Real Numbers**

Classify the following real numbers as rational or irrational:

$$-3, \quad 0, \quad \tfrac{1}{4}, \quad \sqrt{3}, \quad \pi, \quad 7.51, \quad \tfrac{1}{3}, \quad -\tfrac{8}{5}, \quad 6.6666\overline{6}$$

Solution:

Rational: $-3, \quad 0, \quad \frac{1}{4}, \quad 7.51, \quad \frac{1}{3}, \quad -\frac{8}{5}, \quad -6.6666\overline{6}$ Irrational: $\sqrt{3}, \pi$

■ **YOUR TURN** Classify the following real numbers as rational or irrational:

$$-\tfrac{7}{3}, 5.999\overline{9}, 12, 0, -5.27, \sqrt{5}, 2.010010001\ldots$$

■ **Answer:**
Rational: $-\frac{7}{3}, 5.999\overline{9}, 12, 0, -5.27$
Irrational: $\sqrt{5}, 2.010010001\ldots$

Approximations: Rounding and Truncation

Every real number can be represented by a decimal. When a real number is in decimal form, it can be approximated by either *rounding off* or *truncating* to a given decimal place. **Truncation** is "cutting off" or eliminating everything to the right of a certain decimal place. **Rounding** means looking to the right of the specified decimal place and making a judgment. If the digit to the right is greater than or equal to 5, then the specified digit is rounded up, or increased by one unit. If the digit to the right is less than 5, then the specified digit stays the same. In both of these cases all decimal places to the right of the specified place are removed.

EXAMPLE 2 **Approximating Decimals to Two Places**

Approximate 17.368204 to two decimal places by

a. truncation **b.** rounding

Solution:

a. To truncate, eliminate all digits to the right of the 6. $\boxed{17.36}$

b. To round, look to the right of the 6.
Because "8" is greater than 5, round up (add 1 to the 6). $\boxed{17.37}$

Study Tip

When rounding, look to the right of the specified decimal place and use that digit (do not round that digit first). 5.23491 rounded to two decimal places is 5.23 (do not round the 4 to a 5 first).

■ **YOUR TURN** Approximate 23.02492 to two decimal places by

a. truncation **b.** rounding

■ **Answer: a.** Truncation: 23.02
b. Rounding: 23.02

EXAMPLE 3 **Approximating Decimals to Four Places**

Approximate 7.293516 to four decimal places by

a. truncation **b.** rounding

Solution:

The "5" is in the fourth decimal place.

a. To truncate, eliminate all digits to the right of 5. $\boxed{7.2935}$

b. To round, look to the right of the 5.
Because "1" is less than 5, the 5 remains the same. $\boxed{7.2935}$

■ **YOUR TURN** Approximate −2.381865 to four decimal places by

a. truncation **b.** rounding

■ **Answer: a.** Truncation: −2.3818
b. Rounding: −2.3819

It is important to note that *rounding and truncation sometimes yield the same approximation* (Example 3), *but not always* (Example 2).

Order of Operations

Addition, subtraction, multiplication, and division are called arithmetic operations. The results of these operations are called the sum, difference, product, and quotient, respectively. These four operations are summarized in the following table.

OPERATION	NOTATION	RESULT
Addition	$a + b$	Sum
Subtraction	$a - b$	Difference
Multiplication	$a \cdot b$ or ab or $(a)(b)$	Product
Division	$\frac{a}{b}$ or a/b $(b \neq 0)$	Quotient (Ratio)

Since algebra involves *variables* such as x, the traditional multiplication sign $\times$ is not used. Three alternatives are shown in the preceding table. Similarly, the arithmetic sign for division $\div$ is often represented by vertical or slanted fractions.

The symbol $=$ is called the **equal sign**, and is pronounced "equals" or "is," and it implies that the expression on one side of the equal sign is equivalent to (has the same value as) the expression on the other side of the equal sign.

WORDS	MATH
The sum of seven and eleven equals eighteen:	$7 + 11 = 18$
Three times five is fifteen:	$3 \cdot 5 = 15$
Four times six equals twenty-four:	$4(6) = 24$
Eight divided by two is four:	$\frac{8}{2} = 4$
Three subtracted from five is two:	$5 - 3 = 2$

When evaluating expressions involving real numbers, it is important to remember the correct *order of operations*. For example, how do we simplify the expression $3 + 2 \cdot 5$? Do we multiply first and then add, or add first and then multiply? In mathematics, conventional order implies multiplication first, and then addition: $3 + 2 \cdot 5 = 3 + 10 = 13$. Parentheses imply grouping of terms, and the necessary operations should always be performed inside them first. If there are nested parentheses, always start with the innermost parentheses and work your way out. Within parentheses follow the conventional order of operations. Exponents are an important part of order of operations and will be discussed in Section 0.2.

ORDER OF OPERATIONS

1. Start with the innermost parentheses (grouping symbols) and work outward.
2. Perform all indicated multiplications and divisions, working from left to right.
3. Perform all additions and subtractions, working from left to right.

EXAMPLE 4 Simplifying Expressions Using the Correct Order of Operations

Simplify the expressions.

a. $4 + 3 \cdot 2 - 7 \cdot 5 + 6$ **b.** $\dfrac{7 - 6}{2 \cdot 3 + 8}$

Solution (a):

Perform multiplication first. $4 + \underbrace{3 \cdot 2}_{6} - \underbrace{7 \cdot 5}_{35} + 6$

Then perform the indicated additions and subtractions. $= 4 + 6 - 35 + 6 = \boxed{-19}$

Solution (b):

The numerator and the denominator are similar to expressions in parentheses. Simplify these separately first, following the correct order of operations.

Perform multiplication in the denominator first. $\dfrac{7 - 6}{\underbrace{2 \cdot 3}_{6} + 8}$

Then perform subtraction in the numerator and addition in the denominator. $= \dfrac{7 - 6}{6 + 8} = \boxed{\dfrac{1}{14}}$

Answer: a. 10 **b.** $\frac{3}{16}$

YOUR TURN Simplify the expressions.

a. $-7 + 4 \cdot 5 - 2 \cdot 6 + 9$ **b.** $\dfrac{9 - 6}{2 \cdot 5 + 6}$

Parentheses ( ) and brackets [ ] are the typical notations for grouping and are often used interchangeably. When nesting (groups within groups), use parentheses on the innermost and then brackets on the outermost.

EXAMPLE 5 Simplifying Expressions That Involve Grouping Signs Using the Correct Order of Operations

Simplify the expression $3[5 \cdot (4 - 2) - 2 \cdot 7]$.

Solution:

Simplify the inner parentheses. $3[5 \cdot (4 - 2) - 2 \cdot 7] = 3[5 \cdot 2 - 2 \cdot 7]$

Inside the brackets, perform the multiplication $5 \cdot 2 = 10$ and $2 \cdot 7 = 14$. $= 3[10 - 14]$

Inside the brackets, perform the subtraction. $= 3[-4]$

Multiply. $= \boxed{-12}$

Answer: -24

YOUR TURN Simplify the expression $2[-3 \cdot (13 - 5) + 4 \cdot 3]$.

Algebraic Expressions

Everything discussed until now has involved real numbers (explicitly). In algebra, however, numbers are often represented by letters (such as x and y), which are called **variables**. A **constant** is a fixed (known) number such as 5. A **coefficient** is the constant that is multiplied by a variable. Quantities within the *algebraic expression* that are separated by addition or subtraction are referred to as **terms**.

DEFINITION **Algebraic Expression**

An **algebraic expression** is the combination of variables and constants using basic operations such as addition, subtraction, multiplication, and division. Each term is separated by addition or subtraction.

Algebraic Expression	Variable Term	Constant Term	Coefficient
$5x + 3$	$5x$	3	5

When we know the value of the variables, we can **evaluate an algebraic expression** using the **substitution principle**:

Algebraic expression:	$5x + 3$
Value of the variable:	$x = 2$
Substitute $x = 2$:	$5(2) + 3 = 10 + 3 = 13$

EXAMPLE 6 **Evaluating Algebraic Expressions**

Evaluate the algebraic expression $7x + 2$ for $x = 3$.

Solution:

Start with the algebraic expression.	$7x + 2$
Substitute $x = 3$.	$7(3) + 2$
Perform the multiplication.	$= 21 + 2$
Perform the addition.	$= \boxed{23}$

■ **YOUR TURN** Evaluate the algebraic expression $6y + 4$ for $y = 2$.

■ **Answer:** 16

In Example 6, the value for the variable was specified in order for us to evaluate the algebraic expression. What if the value of the variable is not specified; can we simplify an expression like $3(2x - 5y)$? In this case, we cannot subtract $5y$ from $2x$. Instead, we rely on the basic *properties of real numbers*, or the *basic rules of algebra*.

Properties of Real Numbers

You probably already know many properties of real numbers. For example, if you add up four numbers, it does not matter in which order you add them. If you multiply five numbers, it does not matter what order you multiply them. If you add 0 to a real number or multiply a real number by 1, the result yields the original real number. **Basic properties of real numbers** are summarized in the following table. Because these properties are true for variables and algebraic expressions, these properties are often called the **basic rules of algebra**.